APPENDIX 3

OVERVIEW OF A SIMPLE APPROACH TO MODELING INTERNAL LOADING IN LAKE OKEECHOBEE

The Lake Okeechobee internal loading model is a deterministic model that ultimately derives from the original Vollenweider (1975) input-output model used to predict in-lake phosphorus concentrations as a simple function of external loading, hydraulic loss through the lake's outlet, and net sedimentation:

$$V \cdot \frac{dP_{lake}}{dt} = L_{ext} \cdot A - (Q_{out} + \sigma \cdot V) \cdot P_{lake} \tag{1}$$

where P_{lake} is the in-lake phosphorus concentration (mg P/m^3), V is lake volume (m^3), L_{ext} is the loading rate of phosphorus from external sources per unit lake area (mg P/m^2-yr), A is the lake area (m^2), Q_{out} is the lake outflow or discharge rate (m^3/yr), and σ is the *net* sedimentation coefficient (1/yr). σ in essence is a first order decay coefficient that represents the net of two opposing processes: sedimentation by all mechanisms, including gravitational settling of algae and particles, and internal loading (release from sediments), which can occur via a number of processes (Brezonik and Pollman, 1999). Internal loading can or will delay lake recovery, particularly in large, shallow lakes (Welch and Cooke, 1995), and a more appropriate model approach would be to separate σ into two components and consider internal loading and settling losses explicitly (Brezonik and Pollman, 1999):

$$\sigma = \frac{v_{settle}}{z} - \frac{L_{\text{int}} \cdot A}{P_{lake} \cdot V} \tag{2}$$

where $v_{\text{settle}}$is the gross settling or sedimentation velocity (m/yr), z is the mean depth of the lake (m), and L_{int} is areal internal loading or release rate of phosphorus from the sediments (mg P/m^2-yr) . Note that, if the expression for σ in equation (2) is substituted into equation (1), P loss from the system is now simulated as a settling output across the sediment water-interface, rather than as a decay process occurring in the water column.

INCORPORATION OF INTERNAL LOADING INTO A MODEL FOR LAKE OKEECHOBEE

Although a number of mechanisms influence L_{int} in lakes, the two major processes likely to govern the overall internal loading flux of phosphorus in Lake Okeechobee are diffusion and wind-wave generated sediment resuspension. Diffusion here represents the movement of dissolved phosphorus across the sediment-water interface in response to a concentration gradient, and, because the diffusivity describing the mass transfer rate can be enhanced by bioturbation and other physical processes, it typically represents a flux an order magnitude or more greater than simple molecular diffusion. Sediment resuspension is a more stochastic process, and its net affect on water column phosphorus concentrations is dependent upon the amount of labile or exchangeable phosphorus available on resuspended particles, whether dissolved inorganic P concentrations in the water column are above or below the "equilibrium phosphorus concentration" (which dictate whether net adsorption or desorption will occur), and the amount of sediment resuspended (Pollman, 1983).

We can describe a functional relationship for L_{int} in relationship to both water column and sedimentary phosphorus concentrations if we make a series of simplifying assumptions:

(1) We assume that the overall release rate of phosphorus from the sediments is a function of the concentration gradient across the sediment-water interface (Berner, 1983; Lerman, 1979);

(2) We assume that the surficial sediments of Lake Okeechobee are well-mixed over time scales of interest (year), and that the depth of mixing is ca. 5 cm;

(3) We assume that diffusive flux is a product of a mass transfer coefficient (sometimes referred to as a piston velocity) and the difference in dissolved inorganic phosphorus (DIP) concentrations between the porewater concentrations in the surficial sediment mixed layer and the water column.

Water Column Box

The objective of the model in part is to simulate the link between the water column and the sediments, and how this feedback loop from the sediments changes with antecedent conditions in water column. The resultant model is a two box model (water column and sediments) with one compartment in the water column (total phosphorus) and three compartments in the surficial sediments (total sedimentary P, exchangeable P, and porewater DIP; Figure 1). The model maintains mass balance within each compartment. Note that one result of these assumptions is that sediment-resuspension effects are not directly modelled. Although this certainly has implications for predicting short-term dynamics of water column TP and suspended solids, this approximation is probably quite reasonable given that porewater DIP and exchangeable phosphorus concentrations on sediment particles are related by Langmuir-type expressions (Pollman, 1983; Olila and Reddy, 1995) and that the diffusive flux is linearly related to DIP_{pore}.

With the assumptions listed above, we can formulate an explicit expression for the internal loading flux, L_{int}:

$$L_{\text{int}} = v_{diff} \cdot (DIP_{pore} - DIP_{lake}) \tag{3}$$

where v_{diff} is the effective "piston" velocity describing the mass transfer of DIP across the sediment-water interface (m/yr), and DIP_{pore} and DIP_{lake} are the porewater and lakewater DIP concentrations, respectively (mg P/m^3). Rather than introduce another level of complexity into the model, DIP_{lake} concentrations were simulated empirically from the following observed regression relationship observed in Lake Okeechobee (*n* = 3278):

$$DIP_{lake} = -0.0046 + 0.3286 \cdot P_{lake} \qquad r^2 = .48;\ p < 0.0001 \tag{4}$$

DIP_{pore} concentrations are influenced by, in addition to fluxes across the sediment-water interface, sorption exchange with sediment particles, release of DIP from decomposing, organic P in the sediments, and eventual burial below the 5 cm horizon of the active sediment layer.

Equation (1) can now be rewritten by inserting equation (2) and substituting equation (3) for L_{int}:

$$V \cdot \frac{dP_{lake}}{dt} = L_{ext} \cdot A + v_{diff} \cdot \Theta \cdot (DIP_{pore} - DIP_{lake}) \cdot A - (Q_{out} - v_{settle} \cdot A) \cdot P_{lake} \tag{5}$$

where Θ is the porosity of the sediment.

Sediment Box

As mentioned above, the model maintains mass balances for three compartments within the sediments: accreting sedimentary P (other than exchangeable P), exchangeable P sorbed to solid phase particles within the surficial sediments, and DIP. Sedimentary material deposited from the water column is assumed derived from primarily autochthonous sources and is largely organic in nature, with little sorbed P associated with the accreting particles. This is a reasonable assumption since there are large differences in DIP concentrations between the water column and the porewater. The exchangeable P in the sediments thus will be largely derived from the porewater, which in turns derived DIP from the mineralization of deposited organic .

Although sorption is typically represented as an equilibrium process occurring instantaneously, it in reality represents the net balance between the rate of adsorption and desorption. For sorption such as occurs within Lake Okeechobee mud zone sediments and which can be represented by Langmuir-type isotherms, the rate of adsorption is the product of the number of number of sorbent sites available, the concentration of sorbate (in this case, DIP), and the adsorption rate constant (Jaycock and Parfitt, 1981; Pollman, 1983; Morel, 1983):

$$R_{ads} = k_{ads} \cdot DIP_{pore} \cdot (\Gamma_{\infty} - \Gamma) \tag{6}$$

where k_{ads} is the adsorption rate constant, Γ_∞ is the maximum concentration of sorbed phosphorus the solid phase is capable of sorbing, and Γ is concentration of sorbed phosphorus. The total adsorptive flux, J_{ads} is:

$$J_{ads} = k_{ads} \cdot DIP_{pore} \cdot (\Gamma_\infty - \Gamma) \cdot M_{sed} \tag{7}$$

where M_{sed} is the mass of sediment in the surficial mixed layer and is given as:

$$M_{sed} = A \cdot z_{sed} \cdot \frac{(100 - \%H_2O)}{100} \cdot \rho_{bulk} \tag{8}$$

Likewise, the rate of desorption is a first order reaction product of the amount of solute sorbed (Γ) and the rate of desorption:

$$R_{des} = k_{des} \cdot \Gamma \tag{9}$$

The total desorptive flux is:

$$J_{des} = k_{des} \cdot \Gamma \cdot M_{sed} \tag{10}$$

Sedimentary P (organic P) produces DIP through mineralization or decomposition:

$$R_{decomp} = k_{decomp} \cdot P_{sed} \tag{11}$$

Thus, porewater DIP dynamics are expressed by the following equation:

$$\Theta \cdot z_{sed} \cdot A \cdot \frac{dDIP}{dt} = v_{diff}\Theta \cdot A \cdot (DIP_{pore} - DIP_{lake}) + J_{des} - J_{ads}$$

$$+ k_{decomp} \cdot P_{sed} \cdot M_{sed} - v_{burial} \cdot \Theta \cdot A \cdot DIP_{pore} \quad (12)$$

Sedimentary P dynamics integrate gross settling from the water column, burial below the 5 cm mixed layer horizon, and mineralization of organic P to DIP. Rather than simulate sedimentary organic P and total inorganic P fractions explicitly, the entire fraction of settling P is assumed subject to first order decay. The following equation thus represents sedimentary P dynamics:

$$M_{sed} \cdot \frac{dP_{sed}}{dt} = v_{settle} \cdot A \cdot P_{lake} - J_{sedburial} - k_{decomp} \cdot P_{sed} \cdot M_{sed} \quad (13)$$

where the sediment burial flux, $J_{sedburial}$, is given by:

$$J_{sedburial} = \rho_{bulk} \cdot \frac{(100 - \%H_2O)}{100} \cdot 1000 \cdot P_{sed} \cdot A \cdot v_{burial} \quad (14)$$

The third compartment of the sediment box is phosphorus sorbed to sediment particles. This compartment acquires inorganic phosphorus from the pore water via adsorption, and loses phosphorus via desorption and burial. Thus the mass balance equation for sorbed or exchangeable phosphorus on sediment particles is given by:

$$M_{sed} \cdot \frac{d\Gamma}{dt} = J_{ads} - J_{des} - J_{\Gamma burial} \quad (15)$$

where Γ is the concentration of phosphorus sorbed to sediment particles (mg P/kg sediment), and $J_{\Gamma burial}$ is the burial flux of sorbed phosphorus to the deeper sediments:

$$J_{\Gamma burial} = \rho_{bulk} \cdot \frac{(100 - \%H_2O)}{100} \cdot 1000 \cdot \Gamma \cdot A \cdot v_{burial} \quad (16)$$

Sediment Area

Although settling from the water column of suspended material occurs throughout the lake, only portions of the lake bottom serve as true net deposition zones, while other areas of the lake are erosional (cf. Hakanson, 1982). The sediments in Lake Okeechobee are heterogeneous, reflecting the varied depositional environment, with several major sedimentary types (Reddy et al., 1995): mud, peat, littoral marsh, sand/shell/marl, and rock. Of these types, the water content of the mud zone sediments, coupled with their organic matter content, indicate that these sediments comprise the most stable depositional environment. Mesocosm studies by Moore et al. (1994) indicated that internal loading is greatest from mud zone sediments; this is consistent with this zone comprising the most stable depositional environment, thus accreting more organic matter and accordingly producing higher porewater concentrations of DIP.

The model thus assumes that sediment focusing occurs such that all material settling from the water column is transported to the mud zone where it is deposited and remains. As a result, the area of the sediment mud zone, A_{mud} was substituted for lake area, A, as appropriate in equations (5), (8), and (12):

$$V \cdot \frac{dP_{lake}}{dt} = L_{ext} \cdot A + v_{diff} \cdot (DIP_{pore} - DIP_{lake}) \cdot A_{mud} - (Q_{out} - v_{settle} \cdot A) \cdot P_{lake}$$

(5a)

$$M_{sed} = A_{mud} \cdot z_{sed} \cdot \frac{(100 - \%H_2O)}{100} \cdot \rho_{bulk}$$

(8a)

$$\Theta \cdot z_{sed} \cdot A \cdot \frac{dDIP}{dt} = v_{diff} \Theta \cdot A_{mud} \cdot (DIP_{pore} - DIP_{lake}) + J_{des} - J_{ads}$$

$$+ k_{decomp} \cdot P_{sed} \cdot M_{sed} - v_{burial} \cdot \Theta \cdot A \cdot DIP_{pore}$$

(12)

APPLICATION TO LAKE OKEECHOBEE AND MODEL CALIBRATION

The model was applied to Lake Okeechobee using nutrient loading, hydrology, and water column total phosphorus concentrations available for November 1972 through December 1999 (T. James, SFWMD personal communcation). The entire data set was used to conduct the model calibration. Monthly data compiled by James were either averaged or summed as appropriate to produce annual values. For example, all flows and fluxes were summed to produce the total annual flow or flux. Concentrations and lake area and volume were averaged to produce yearly values. Predicted annual average lakewater concentrations were then compared to observed values as part of the calibration. Although James previously had computed nutrient input fluxes from atmospheric deposition, these estimates were revised to reflect the best available data on measured wet deposition fluxes in south Florida (Pollman and Landing, unpublished data), and estimates of likely dry deposition inputs (LO TAC, May 2000). These estimates equate to an annual average total deposition flux of 18 mg/m^2-yr. Annual fluxes input to the model were scaled to reflect the relative year-to-year variations James had compiled for the lake, but still produce the same average annual flux across the period of record.

Table 1 lists the input and calibrated values for all the parameters in the model, while Table 2 lists the initial values for the various phosphorus compartments in the model. Physical aspects of the mud zone sediments (% water, particle density, bulk density) were obtained from Pollman (1983). Percent water content of surficial sediments ranged from ca. 82 to 91%; a value of 90% was selected for the model. Likewise, a particle density of 1.5 g/cm^3 was used in the model consistent with the water and organic matter content of the mud zone sediments. Sorption parameters for mud zone sediments were obtained from Pollman (1983), who studied adsorption and desorption for several different substrate types in Lake Okeechobee.

Table 1. Parameter values for the Lake Okeechobee internal loading phosphorus model.

Parameter	Units	Value	Description	Source
ρ	g/cm^3	1.5	sediment particle density	Pollman (1983)
$\%H_2O$	%	90	percent water in sediment	Pollman (1983)
ρ_{bulk}	g/cm^3	Calculated from ρ and $\%H_2O$	sediment bulk density	
Θ	dimensionless	Calculated from ρ and $\%H_2O$	fraction of sediment volume occupied by void	
z_{sed}	m	0.05	depth of sediment mixed layer	Data from Reddy et al. (1991)
k_{decomp}	1/yr	0.2	sedimentary organic P decomposition rate	calibrated
k_{des}	1/yr	1	inorganic P desorption rate	Pollman (1983)
k_{ads}	mg/kg-yr	0.0075		Pollman (1983)
Γ_∞	mg/kg		Langmuir adsorption capacity	Pollman (1983)

v_{settle}	m/yr	8.229	gross deposition velocity from the water column to the sediments	calibrated
v_{diff}	m/yr	11.888	diffusion “piston” velocity	calibrated
v_{burial}	m/yr	Variable, range from 0.00096 to 0.0029, average 0.00235	burial velocity of sediments and porewater	Brezonik and Engstrom (1998)
DIP_{lake}	mg/m^3	calculated	In-lake dissolved inorganic P	Regression relationship with TP from observed data, 1973 -1999
A_{sed}	m^2	Allowed to vary from 503.2 x 10^6 in 1975 to 740 x 10^6 in 1988	area of sediment mud zone	Havens and James (1999)

Sediment burial rates in Lake Okeechobee have changed in time, reflecting presumably changing trophic state and differing rates of internal production and external inputs of particulate matter. For example, between 1940 and 1988, the average v_{burial} for mud zone sediments was 0.00345 ± 0.00205 m/yr compared to 0.00186 ± 0.00110 m/yr for intervals predating 1940 (Brezonik and Engstrom, 1998). v_{burial} was then changed from a constant and allowed to vary as a first order function of the gross settling flux of particles from the water column. Scaling was based on simulated variations in gross sedimentation flux to produce v_{burial} rates over the simulation period that varied linearly (proportionally) with changes in the gross sedimentation flux and still produced a specified mean value. The final calibrated average value of 0.00235 m/yr is well within the range of values reported by Brezonik and Engstrom (1988) and closely approximates the median 1940 – 1988 value of 0.00208 m/yr.

Table 2. Initial values for phosphorus components in Lake Okeechobee internal loading phosphorus model.

Parameter	Units	Description	Value
P_{lake}	mg/m^3	In-lake total P concentration	41.5
DIP_{lake}	mg/m^3	In-lake dissolved inorganic P concentration	13.6
DIP_{pore}	mg/m^3	In-lake dissolved inorganic	75

		P concentration	
P_{sed}	mg/kg	Sedimentary TP concentration	850
Γ	mg/kg	Sediment sorbed P concentration	25

The rate constant describing the decomposition of sedimentary organic P, k_{decomp}, was calibrated by comparing resulting predicted P_{sed} concentrations with observed values. This rate was 0.2/yr. Figure 2 shows the results of the predicted P_{sed} concentrations for the period of record, which range from an initial value of 850 mg/kg to a final predicted value of 1448 mg/kg. These results are consistent with the overall increasing trends in sedimentary P noted by Brezonik and Engstrom (1988).

Annual average lakewater TP concentrations predicted by the calibrated model are compared in Figure 3 with observed observed annual average lakewater concentrations measured by James (personal communication). The agreement is reasonably good, with a coefficient of determination of 0.52. As evidenced by the somewhat lower standard deviation, the model does not quite capture the full range of variability in the observed data (Table 3). This is not surprising, given the comparative simplicity of the model, and lack of stochastic inputs to the model such as wind-driven sediment resuspension. Nonetheless, consistent with the objectives for which the model was developed, the model does appear to perform well in terms of long-term predictions. There is essentially no overall bias in the model, with an average predicted concentration over the period of record of 87.4 mg/m^3 compared to an observed average of 87.5 mg/m^3.

Table 3. Comparison of statistical properties of model predicted annual average TP concentrations with observed concentrations, 1973 – 1999. All concentrations in mg/m^3.

Statistical Metric	Observed	Predicted
Mean	87.5	87.4
Median	89.2	90.4
Standard deviation	19.8	13.7
97.5% quantile	115.7	105.7
2.5% quantile	49.7	49.6

SENSITIVITY ANALYSES

It is important to understand which parameters in the model have the greatest influence on the predictive behavior of the model. A standard approach is to conduct a sensitivity analysis where input parameters are changed by a given amount, and the resultant changes in model predictions are evaluated. For this analysis, sensitivity analysis was applied to all the rate coefficients and parameters listed in Table 1. Each parameter was varied by ± 20% except for %H_2O, where such a change did not make physical sense and sensitivity to changes of ± 10% were evaluated. Because the model response to changing loads is somewhat non-linear, the sensitivity analysis was conducted under two loading regimes: the first under "current" long-term average conditions; the second under the estimated TMDL load reduction of 75.6% required by the calibrated model to achieve a long-term average water column TP concentration of 40 mg/m^3 (see following section). Loadings under both regimes were kept constant throughout the simulation. Model response was then gauged at $t = 200$ years – *i.e.,* well after the model had essentially reached steady-state.

The model endpoint selected for analysis was P_{lake}. To allow a common basis to compare the effects of different inputs, results were normalized to the relative rather than absolute response:

$$\Psi_{rel} = \frac{\frac{(P_\infty - P_{lake})}{P_{lake}}}{\frac{\lambda - \lambda_{cal}}{\lambda_{cal}}}$$

where Ψ_{rel} is the normalized response, P_∞ is the predicted steady state lakewater TP concentration, λ_{cal} is the parameter value used in the calibrated model, and λ is the perturbed value.

It should be noted that this type of traditional sensitivity analysis does not account for the actual variability/uncertainty associated with each input parameter. Some inputs may vary proportionately more than others. Moreover, it also does not account for the fact that some inputs may co-vary, such as ρ, %H_2O, and, in shallow lakes such as Lake Okeechobee, z_{sed}. These limitations can be accommodated via a Monte Carlo approach that uses probabilistic distributions for key inputs (assuming these distributions are known) and includes relationships between inputs, but is beyond the scope of the current exercise.

The results of the sensitivity analysis are shown in Figures 4 and 5. The results show that the model is most sensitive to uncertainty in parameters that relate to the flux of phosphorus to and from the sediments and the turnover rate of phosphorus in the sediments (v_{settle}, v_{burial}, z_{sed}, k_{decomp}). The model also is sensitive to uncertainty in A_{sed}, which influences both the concentration of phosphorus in the sediment and the flux of phosphorus from the sediment to the water column and, to a lesser extent, %H_2O, which affects principally the amount of phosphorus stored in the sediments. Of the parameters judged most sensitive, k_{decomp} and v_{settle} were the only two fitted variables for which no direct empirical data were available for parameterization.

Uncertainty in the phosphorus residence time in the surficial sediments will influence the predicted response time to changes in external loads as well as P_{∞}. Figure 6 shows the predicted magnitude and rate of response in P_{lake} to stepped changes in load as a function of z_{sed}, which was allowed to vary ± 20%. The simulation was conducted by taking the model response at the end of 1999 and equilibrating the model for 10 years to long-term average (constant) loads observed between 1973 and 1999. This was done to eliminate any transient response in the model during the early phase of the simulation because the system was not at steady state. The model was then run for an additional 30 years (i.e., to $t = 40$ years) with a stepped load reduction of 75.6% occurring at the end of year 10. Under these conditions, the calibrated model produces a steady state response of 40 mg/m^3. The upper panel of Figure 6 shows the actual magnitude of the concentration changes for given changes in z_{sed}; the lower panel shows the fractional response (relative to the total change in P_{lake} as the system moves from the initial conditions at the start of the load reduction to P_{∞}). All three scenarios show nearly identical responses during the first 2 – 3 years following the load reduction; response during this period is governed by the lake hydraulic residence time and the magnitude of the load reduction. Beyond this initial period of rapid response (which produces *ca*. 30% of the ultimate steady state response), internal loading from the sediments dominates. During this latter phase, the response curves for each scenario diverge, with the most rapid response corresponding to the scenario with the shallowest z_{sed} and thus the shortest sedimentary P residence times. Times to reach 90 % of the total response ranged from 27.6 to 34.4 years. The 90% response of the calibrated model was 31.2 years.

LONG-TERM LOAD-RESPONSE CALCULATIONS

Long-term predictions and TMDL limits were calculated using long-term average hydrologic and nutrient loading conditions observed in Lake Okeechobee between 1973 and 1999. Table 4 summarizes these parameters, as well as summarizing the predicted P_{∞} concentration should these loading and hydrologic conditions continue. Under the estimated long-term hydrologic regime, the acceptable load limit that is predicted to result in a long-term average concentration of 40 mg/m^3 is 121 tonnes/yr. This equates to a 75.6% reduction in the current long-term loading rate of 496 tonnes/yr. The response between changing loading rates and P_{∞} is non-linear; the model predicts that the efficiency[1] of the lake to retain sedimentary P declines as water column concentrations decrease (Figure 7). This effect is the result of a greater relative amount of mineralization of organic P occurring in the sediments as the flux of settling detritus declines with declining P_{lake} and the residence time of organic P in the surficial sediments increases accordingly.

The TMDL load calculation assumes that the load reduction comes at the expense of changing inflow concentrations to the lake, with no change in the overall hydrology. Changing the hydrology to achieve the requisite load reduction will change the hydraulic residence time of the lake, and thus its assimilative capacity. This is illustrated in Table 4, which also includes predicted P_∞ and TMDL loads for two scenarios with changes in the inflow rates to the lake. The two scenarios consider increases and decreases in the hydraulic loading rate, q_s (m/yr; defined by Q_{in}/A), by 20%; inflow nutrient concentrations are adjusted accordingly to produce the same long-term load of 496 tonnes used in the original TMDL calculations. It is unlikely the evaporative losses from the lake surface would change appreciably in response to the imposed changes in q_s; hydrologic balance thus was maintained by keeping evaporation constant and adjusting Q_{out}. If q_s is increased by 20% to 0.66 m/yr, then P_∞ for a long-term loading regime of 496 tonnes/yr declines from 105 to 101 mg/m^3 and the TMDL load increases from 121 to 134 tonnes/yr. Reducing q_s to 0.44 m/yr results in a higher P_∞ and a lower TMDL load (114 mg/m^3 and 101 tonnes/yr, respectively).

Table 4. Comparison of P_∞ concentrations and threshold TMDL loads for given changes in assumed long-term hydraulic loading rate, q_s. Analysis assumes that evaporative losses from the lake are unaffected by q_s and remain constant; hydrologic balance is thus maintained by adjusting the outflow rate correspondingly.

	Change in Hydraulic Load		
Parameter	0	+20%	-20%
Area (m2/yr)	4.65E+09	4.65E+09	4.65E+09
Inflow (m3/yr)[1]	2.56698E+09	3.08038E+09	2.05358E+09
Outflow (m3/yr)	1.68978E+09	2.20318E+09	1.17638E+09
Inferred Evaporation (m3/yr)[1]	8.77200E+08	8.77200E+08	8.77200E+08
Inflow Concentration (mg/m3)	193.31	161.09	241.64
Load (tonnes)	496.2	496.2	496.2
q_s (m/yr)[1]	0.55	0.66	0.44
Evap oration (m/yr)	0.19	0.19	0.19

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[1] Efficiency of sediment P retention can be estimated by calculating the net settling velocity, v_{net}(m/yr), where $v_{net} = \sigma/z$. When substituted in the Vollenweider input-output model, Equation 1 thus becomes:

$$V \cdot \frac{dP_{lake}}{dt} = L_{ext} \cdot A - (Q_{out} + v_{net} \cdot A) \cdot P_{lake}$$

P_∞ (mg/m^3) (current load)	105	101	114
TMDL Coefficient[2]	0.244	0.271	0.203
TMDL Load (tonnes/yr)[3]	121	134	101
P_∞ (mg/m^3) (at load of 121 tonnes)	40	37	45

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[1] Direct precipitation to the lake surface has been excluded (although the load has been included); thus both q_s and inferred evaporation, which is calculated by difference from Q_{in} and Q_{out}, reflect this exclusion. Since flushing is governed by the outflow rate, this simplification has no effect on the model calculations.

[2] Reduction coefficient applied to current long-term average load necessary to produce a steady state lakewater TP concentration (P_∞) of 40 mg/m^3.

[2] External TP loading rate required to produce a steady state lakewater TP concentration (P_∞) of 40 mg/m^3. TMDL Load = $\text{Load}_{\text{current}}$ x TMDL Coefficient.

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